CTmathU Computational Thinking in University Mathematics

Computational Thinking; Math Inquiry; Teacher Education; Instrumental Genesis ctuniversitymath.ca

Coding in math learning: A 'triple instrumental genesis' approach to support the transition from university learner to school teacher

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There is a pressing need to rethink the preparation of math teachers for their integration of computational thinking (or coding)

- We are witnessing a rapidly changing landscape of coding and computational thinking integration in compulsory education in many parts of the world; e.g. in France, England, Sweden, Australia, and Canada.
- PISA, in their 2022 math assessment framework, states that "students should possess and be able to demonstrate computational thinking skills as they apply to mathematics" and that they anticipate "a reflection by participating countries on the role of computational thinking in mathematics curricula" (OECD, 2018, p. 5, para. 12).
- Since 2001, the Mathematics Department at Brock University in Ontario (Canada) has implemented a sequence of three courses, *Mathematics Integrated with Computers and Applications* (**MICA I-II-III**), in which math students (including future math teachers) learn to use coding to investigate mathematical concepts, conjectures, theorems, and applications.
- In 2020-21: coding was integrated in Ontario math curriculum, which created a need to **revisit the design of the MICA III section dedicated to future teachers.**

Coding in the Ontario Math Curriculum in Grades 1-8 (2020)

Expectations (in the Algebra Strand)	....	Grade
Solve problems and create computations of mathematical situations by writing and executing code, include code that involves .... + read and alter existing code, including code that involves ...., and describe how changes to the code affect the outcomes	Sequential events	1
	Concurrent events	2
	Repeating events	3
	Nested events	4
	Control structures (e.g. conditional statements)	5
	Efficient coding	6
	Events influenced by a defined count and/or sub-procedures	7
	Analysis of data in order to inform and communicate decisions	8

Ontario Math Curriculum for Grade 9 (2021)

Coding (in the Algebra Strand): with the goal to "dynamically represent situations, analyse mathematics concepts, and solve problems in various contexts"

Implementation at Brock University since 2001

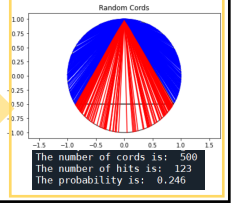
2 hrs lecture + 2 hrs lab, weekly (Buteau et al., 2015)

MICA Inquiry Projects (~75% of final grade)

Year	Project
MICA I For all	Conjecture about primes or hailstone sequence
	RSA encryption method
	Discrete dynamical system (cubic with two parameters)
	Original project (individual or in pair)
MICA II For all	Buffon needle problem & Monte Carlo integration
	Markov chains applied to income demographics and chronic illness & Statistical application to stock market
	Bifurcation diagram
	Battle simulation (Lanchester equations)
	Original project (individual or in pair)
MICA III For future teachers	Gregorian calendar problem in Scratch
	Random chords in a circle Problem
	Predator-prey model (Lotka-Volterra equations)
	Randomness of DNA sequences
	Original collaborative project with a local teacher

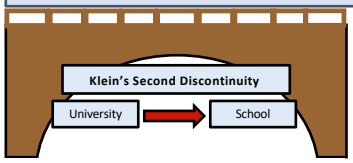
What is the probability that a random cord in a circle be longer than the side of an inscribed equilateral triangle?

```
#the 0.25 probability case
def FirstExample():
    count = 0
    for i in range(0, numcords):
        y=2*random.random()-1
        x=np.sqrt(1-y*y)
        z=random.random()
        if z<0.5:
            x=-x
        if y<-0.5:
            RedLine(x,y)
            count = count + 1
        else:
            BlueLine(x,y)
    return count
```




```
The number of cords is:  500
The number of hits is:  123
The probability is:  0.246
```

Klein's Double Discontinuity and Our Research Question



The challenges faced by teachers in the transition from university math learning to school math teaching have been known for a long time and have been addressed by the INDRUM community (see, e.g., an ongoing international seminar; Grenier-Boley, n.d.).

To address the second of "Klein's double discontinuity", we based our redesign on the following research question:

QUESTION: How can a mathematics content course for future teachers assist them in gaining skills and attitudes needed for making the transition to their future role of teachers, specifically in the case of using coding for mathematics learning?

Redesign of MICA III course for future teachers

Used an "experiential" education (Kolb, 1984) perspective to provide opportunities for future teachers to make productive links between their learning of university math and their future profession as school teachers.

Learning Objectives:

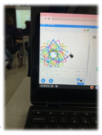
O1) to further one's experience of using coding to learn math (including conducting investigations);

O2) to develop an understanding of that experience (the learning), including affordances of coding for math; and

O3) to develop an understanding of teaching (supporting the learning) of using coding to learn math and curriculum.

Main Activities:

- The course continues to be structured around four individual coding-based math inquiry projects (similar to MICA I-II; **O1**).
- Each inquiry project is complemented with posteriori revised guided reflections based on selected new readings (**O2,O3**).
- Two new lab activities were introduced on learning and comparing coding languages (**O2,O3**).
- A collaborative project was revised, where future teachers work in pairs **with a local school teacher** to prepare and implement a coding and math activity in the school classroom, and reflect on the experience (**O3**). For videos and reports, see: mkn-rcm.ca/niagara-catholic-brock-u-collaborative-coding/



Exploring framing learning with a triple instrumental genesis approach?

This study is part of a larger research project (2017-2023) examining the learning and teaching of using programming as an instrument for pure and applied mathematical inquiry projects in university education (ctuniversitymath.ca). We use the instrumental approach (Rabardel, 1995) in which programming is seen as an artefact (human product) that may be transformed into a meaningful instrument (by developing 'schemes'), through a process called 'instrumental genesis'.

In this preliminary study, we are proposing that future teachers undergo:

→ a *personal genesis:* develop schemes to use coding in their own math learning (Gueudet et al., 2020): O1

→ a *professional genesis* (Haspekian,2011): develop schemes to use coding for didactic purposes in math classrooms: O3

→ as part of the latter, the teacher must also support *school students' geneses* of coding for math learning: O1, O2, O3.

Examples of Future Teachers' Reflections (Data Excerpts)

Identify a 'key' strategy used by the instructor, TA, or a peer helper to support your completion of the project and the ways in which it supported your work or learning. "A 'key' strategy my TA used was writing it out on paper initially, then figuring out how to encode it in a way that the program would understand. Doing the math out by hand first helped me understand what boundaries would need to be set which made it easier to generate an idea of how to code it… This will help me with future projects…" (S#10)

Consider the cognitive, interpersonal, and intra-personal domains (Vivian et al., 2020). Illustrate how each domain was part of your experience in completing this project. "A key moment … when I encountered difficulties trying to comprehend the DNA sequence of normalised walks and how to program the code…[I] asked a peer who… first asked me if I understood mathematically what was required and if I could first explain… in words what I wanted my program to do. Initially, I struggled …and this is what led me to… self-reflect that the problem was that I was not understanding what I wanted my program to do and this is why I encountered many errors/bugs …" (S#1)

"I just find that I wasn't given enough information to do it on my own and feel completely overwhelmed" (S#2)

"At the start … you could see … my frustration and how… I was saying … we should have been provided more … how to start, how to continue…But over time… was a lot of me reflecting as a learner and saying… I ran into this problem, but this is how I solved it. I didn't use a lot of my professor's assistance…"

"now I'm gonna be the 'Buteau' to my Gr. 9 class… not telling students as much as you want to… guide them to the path." (S#1)

Next Steps

- Refine our conceptualization of future teachers' learning (Sacristán et al., 2022)
- Analyse student data (student work, perceptions collected through pre-/post-questionnaires, interviews) to evaluate the course design
- prepare for the next iteration of course design refinement.

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