Integrating Coding in Mathematics: Effectiveness of A Project-Based Approach

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INTRODUCTION

- In a recent "Call for Research that Explores Relationships between Computing and Mathematical Thinking and Activity in RUME," Lockwood and Mørken (2021) suggest that different approaches provide "opportunities for systematically studying different ways for [the] integration [of computing] to occur" (p. 6).
- They point out, in particular: "there are an increasing number of examples of meaningfully-integrated programs across the world, and the RUME community can explore what kinds of programs are effective and why".
- In this study, we address the above call by **examining students' learning as they engage in programming for mathematics investigations through a Project-Based Learning (PBL) approach**. In particular, the study focused on the effectiveness of a sequence of 14 programming-based mathematics projects, as reported by 41 students.

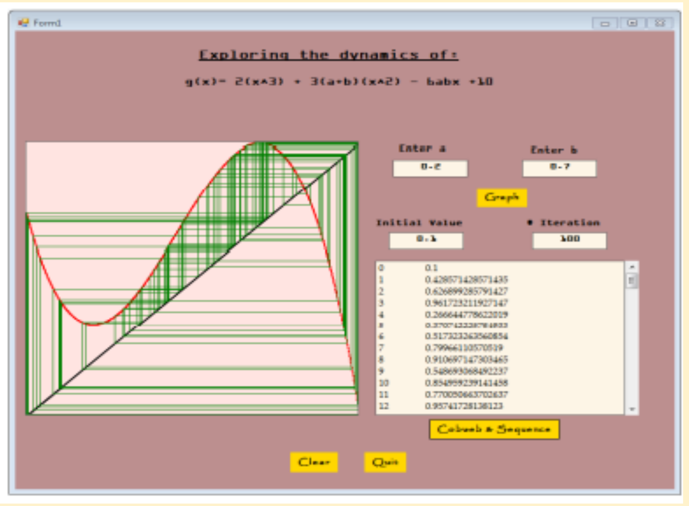
THEORETICAL FRAMEWORK

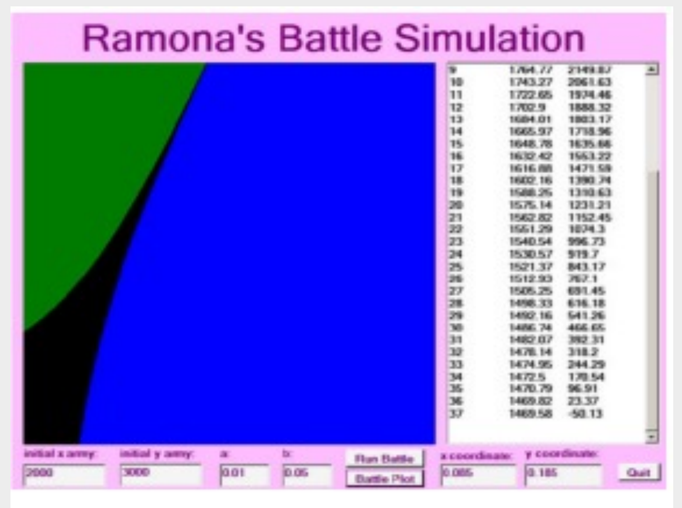
- Framework which specifically guides this work is the **constructionist paradigm** (Papert & Harel, 1991) which embodies a particular kind of **PBL approach** in which students consciously and actively engage in constructing (e.g., through programming) tangible and shareable objects.
- Given our context of a technology-rich environment, we further frame "**student learning**" using the notion of **instrumental genesis** (Guin et al., 2005; as described in Buteau et al., 2019), conceptualized as a complex process involving the intertwinement of learning techniques for using a digital tool and learning specific mathematics concepts (Artigue, 2002).
- Using Vergnaud's (2009) theory, this process can be seen as potentially involving the learning of two kinds of knowledge: **operational knowledge** (which provides means to do and succeed) and **predicative knowledge** (which consists of means to express ideas in words or symbols).

CLASSROOM CONTEXT

- Our work is contextualised in **three specially designed project-based courses** called Mathematics Integrated with Computers and Applications (MICA) I, II, and III at Brock University, which **engage students in using programming for authentic pure and applied mathematics investigations** (Buteau, Muller, & Ralph, 2015)

E.g. Students design and program an environment to explore, graphically and numerically, the dynamics of the dynamical system based on a 2-parameter cubic function (P3)





E.g. Students design and program an environment to explore a model of battles between two armies and simulate different situations, include the random addition of troops throughout the battle (P8)

METHODOLOGY

- **Participants:** 41 volunteer students from MICA I, II and III
- **Data:** Students' responses to a post-course online questionnaire, including asking which project they learned the most from, and why (multiple choice)
- **Data analysis:** Participants' responses (which project?) were interpreted as indicating the "most effective" project.
- We reflected on the kinds of projects that were selected:
  - pure math project,
  - applied math project, or
  - passion project (they select their own topic)
- Students' responses to 'why?' were analysed using frequency graphs, and also separately according to the type of project.
- We also regrouped students' reasons into two categories based on whether we interpreted them as indicative of learning predicative knowledge or learning operational knowledge.

RESULTS

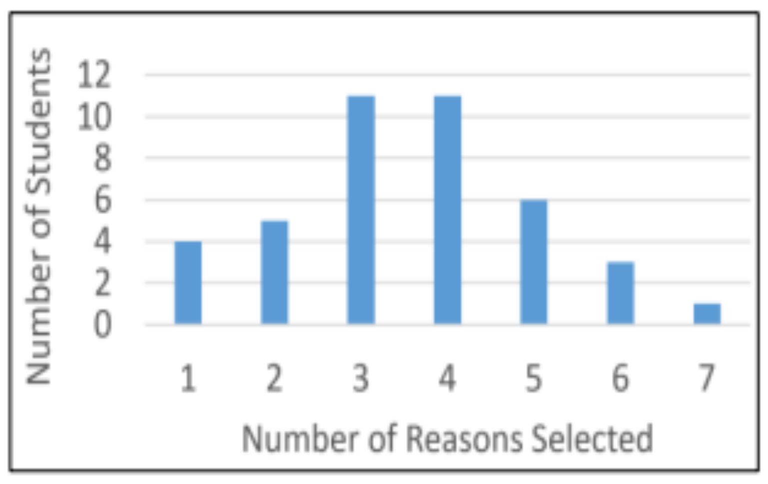
- Students **selected a variety of projects** : pure and applied math; some have a higher ceiling; differ in the degree to which projects are scaffolded by instructors ; projects at different moments in the sequence of 14.
- Among the projects that were selected, nearly half (20/41) were **Passion Projects ('PP')**.

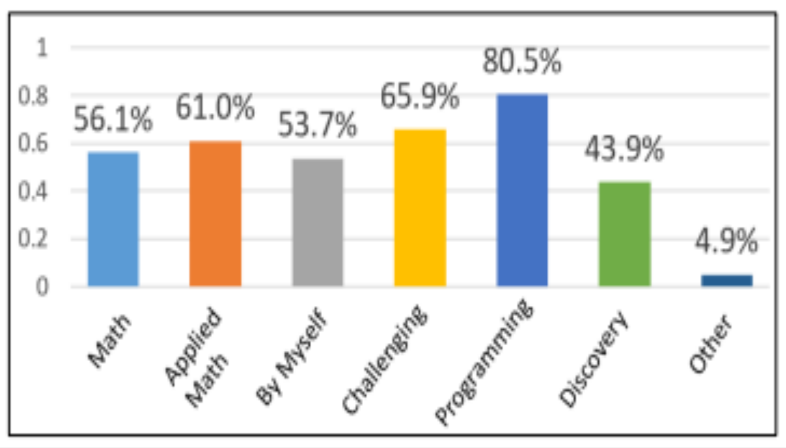
  → This provides empirical evidence in support of certain constructionist claims: e.g., that students learn best when they are working on projects that are meaningful to them (Papert, 1980), i.e., on topics that they are passionate about (Resnick, 2014).
- When looking at MICA I participants only, **Project 2 (on RSA)** was the most selected (41%). We note that it is the **first project** where students must **program a more elaborate mathematical process**, in addition to using new programming concepts and skills.

Course	P1	P2	P3	PP4	P5	P6	P7	P8	PP9	P10	P11	P12	P13	PP14
MICA I	1	10	5	8	-	-	-	-	-	-	-	-	-	-
MICA II	0	0	0	1	0	0	0	1	3	-	-	-	-	-
MICA III	0	1	0	2	0	0	0	1	2	0	1	0	1	4
Total	**1**	**11**	**5**	**11**	**0**	**0**	**0**	**2**	**5**	**0**	**1**	**0**	**1**	**4**

Table 1. The projects participants said they learned the most from.

Year	Project	Type	Topic
1 *for all*	P1	Pure	Conjecture about primes or hailstone sequence
	P2	Applied	RSA encryption method
	P3	Pure	Discrete dynamical system (cubic with two parameters)
	P4	Passion	End of term project
2 *for all*	P5	Pure	Buffon needle problem & Monte Carlo integration
	P6	Applied	Markov chains applied to income demographics and chronic illness & Statistical application to stock market
	P7	Pure	Bifurcation diagram
	P8	Applied	Battle Simulation (Lanchester equations)
	P9	Passion	End of term project
3 *for future teachers*	P10	*Exercise*	Calendar problem in Scratch
	P11	Pure	Simulations about Bertrand's Paradox
	P12	Applied	Predator-prey model (Lotka-Volterra equations)
	P13	Applied	Randomness of DNA sequences
	P14	Passion	End of term project





- Nearly 78% selected at least 3 reasons

  →This **highlights the richness of the students' learning experiences** in their chosen projects.
- "I had to use new programming concepts or skills" was selected the most (80.5%).

  → This shows that as students engage in these projects, they typically encounter avenues where in addition to applying their prior knowledge in programming, they also have to use new knowledge to create their program and complete their projects.
- Two 'Other' reasons (both for PPs):
  - "I was paired with a cool partner, and we worked well together" (**'Peers'**)
  - [opportunity to] "create a lesson plan which incorporates programming to teach mathematics to grade 9 students." ('**Passion**')

Table 2. The kind of knowledge students learned in their chosen projects.

Projects	Operational & Predicative	Only Operational	Only Predicative
All	30	8	3
PP	13	5	2
P2	9	2	0

- It appears that over **70%** of participants learned both operational and predicative knowledge in the project they learned the most from. This reflects the different kinds of knowledge required in using programming for mathematics like mathematicians do (Broley, 2015).
- It aligns with the PBL approach which engage students in learning the general processes and attitudes involved in (mathematical) problem solving, in addition to learning specific (math) content.

Predicative Knowledge (reflecting

- "I learned a lot of new math"
- "I learned a real-world math application that really speaks to me"
- "I discovered something I did not expect"

Operational Knowledge

- "I completed it all by myself without help"
- "It was challenging but I finally understood it"
- "I had to use new programming concepts or skills"

CONCLUSION

- **Contribution to answering Lockwood and Mørken's (2021) call for research** by exploring the **effectiveness of one approach to integrating computing in undergraduate mathematics**: namely, **PBL**, as defined by general work in education
- **Contribution to addressing the gap of empirical evidence related to the effectiveness of such a PBL approach** (e.g., reporting on the different kinds of learning that may occur in programming-based mathematics investigation projects).

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