Mathematics Integrated with Computers and Applications II

An Undergraduate Programming-Based Mathematics Course for Mathematics Majors and Prospective Teachers, Brock University (Canada)

Five Programming-Based Math Project Assignments

Dr. Bill Ralph (Brock University)

Since 2001, mathematics majors and prospective mathematics teachers learn through a sequence of three courses (MICA I-II-III) to design, program and use interactive environments for the investigation of math concepts, conjectures, theorems and applications. This is the second course in which students are expected to have learned the basics of programming (in vb.net) and of programming for mathematical investigations.

This MICA II course leads to a third one whereby students are then divided into two sections; one for mathematics and science majors (MICA III) and one for prospective mathematics teachers (MICA III*).

Programming Languages:

- *Visual Basic.NET* through *Visual Studio* development environment (this is the same as in MICA I)
- Excel (in assignment 2 only)

For some information about the MICA courses, see:

- Ralph, W. (2001). Mathematics takes an exciting new direction with MICA program. *Brock Teaching,* 1(1), 1.
- Buteau, C., Muller, E., & Ralph, W. (2015). Integration of Programming in the Undergraduate Mathematics Program at Brock University. In Online Proceedings of the *Math + Coding Symposium,* London (Canada), June 2015.Acessible at: http://researchideas.ca/coding/docs/ButeauMullerRalph-Coding+MathProceedings-FINAL.pdf

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Here are some examples of the types of projects students have completed over the years in MICA II.

- Find the hypervolume of a hypersphere. (Monte Carlo Techniques)
- Analyze the stock market and pick the “best” stocks. (Random Variables, Histogram, Mean, Standard Deviation, Skewness, Regression and Correlation)
- Look for mathematical patterns in DNA sequences. (Bioinformatics)
- Model changing income demographics. (Markov Chains)
- Zoom in on bifurcations in chaotic systems. (Discrete Dynamical Systems)
- Simulate battles between armies. (Stochastic Processes and Differential Equations)
- Find the balance in a predator-prey biological system. (Coupled Differential Equations, Euler’s Method, Phase Plane Trajectory)
- Examine the effects of inoculation on the spread of epidemics. (Cellular Automata, Differential Equations)
- Model of water pollution in a system of two connected lakes connected by a stream (Markov Chains)
- Examine bacterial growth (stochastic models)

Additional details at: http://researchideas.ca/coding/docs/ButeauMullerRalph-Coding+MathProceedings-FINAL.pdf

Winter 2019 MICA II Assignment #1: Monte Carlo Integration Applications

Task guidelines created by Bill Ralph (Brock University)

Brock University

Note: All of your code should be carefully structured and very easy to read with all variables, functions and subroutines labeled in a helpful way. In addition, the interface should be user friendly and attractive.

1) Suppose that a needle of length ½ is dropped onto a plane of parallel lines that are 1 unit apart. By modifying the Buffon Needle program given in class, find the probability that the needle touches a line. Hand in your finished program which should look like the one given in class but with the appropriate modifications. Label this program as “Buffon Needle Problem”. (25 marks)

2) Consider the region R in [0,2] x [0,2] for which

   sin(a*x*x + b*y*y*y) > sin(c*x*x + d*y*y)

   Hand in a program that makes this area appear on the screen for different values of a,b,c and d and estimates its area using n points chosen at random in R. The user should be able to input a,b,c,d and n. Label this program as “Area In A Square”. (25 marks)

3) By choosing n points at random inside $[-1,1]^4$, write a program to estimate the hypervolume of the unit hypersphere in R^4 which is the set of points for which $x^2+y^2+z^2+w^2 <= 1$. The user should be able to input the sample size n and the number of samples w. The output should show the mean and standard deviation of the w samples. Estimate the hypervolume accurate to one decimal place and use your observations to explain why you are confident that your first decimal place is correct. Also hand in a printout of your code which should be in the simplest possible form. Do not hand in the working program. (25 marks)

Do **either** question (4) or question (5). Your choice!

4) Suppose that a needle of length 1 is dropped onto a plane of parallel horizontal and vertical lines that are 1 unit apart. By modifying the code for the Buffon Needle program given in class, find the probability that the needle touches any of the lines. Hand in your written explanation of the mathematics behind your method as well as your working program. This program does not have to have a graphical component (unless you’d enjoy giving it one). Label this program “Buffon-Laplace Problem”. (25 marks)

5) Suppose that two numbers a and b are chosen at random from {1, 2, …, n }. Let P_n be the probability that they are relatively prime. As n goes to infinity, does the limit of P_n exist? Hand in the program you write to investigate this question and a discussion of what you observed. Can you guess the **exact** limit? (25 marks)

Winter 2019 MICA II Assignment #2

Task guidelines created by Bill Ralph (Brock University)

Note: use type double for the number of people in each category (in other words, we will allow fractions of a person).

1) Suppose a certain chronic illness has three distinct stages of severity that change from year to year as follows. If someone is in Stage I, the most benign state, there is a 90% chance of remaining there, and a 10% chance of progressing to the intermediate State II. If someone is in Stage II there is an 80% chance of staying there, a 10% chance of going on to the most severe Stage III, but another 10% chance of returning to Stage I. If someone is in Stage III there is an 80% chance of remaining there, and a 20% chance of returning to Stage II. Suppose there are initially 10000 people at each stage.
   a) By squaring a matrix, find the number of people in Stage III after two years.
   b) Write a program that finds the number of people at each of the three stages after a very long time has passed. Your program should include a graph that shows the number of people in each category in each year.
   c) Verify the equilibrium you found in (b) by doing a hand calculation.

2) Suppose current socioeconomic data reveals that 50% of children whose parents were low-income wage earners will also be low-income earners, 40% of these children will instead be middle-income earners and another 10% will reach high-income earnings. For middle-income parents, 80% of their children will also be middle-income earners, but 10% will be low-income earners and another 10% will be high-income earners. For high-income parents, 70% of their children will also be high-income earners, but 25% will be middle-income earners and 5% will be low-income earners.
   a) Suppose that the total number of people in the three categories is always 300000. Modify your program from part 1b to investigate whether the steady-state of this system depends upon how many people were in each category initially. Write down your conclusion and all the different equilibria that you find.
   b) Modify your program in 2a so that children of high-income earners are always high-income earners. What happens after a long time? Give evidence.
   c) Give a complete mathematical proof to support your observation in 2b.

3) Go to Yahoo Finance and download an Excel sheet containg the daily closing prices of the Dow Jones from 1950 until now. Use Excel to create a column containing the daily return percentages given by $100\,(X_{n+1} - X_n)/X_n$.
   a) Use Excel to construct a histogram of the daily return percentages. Is it symmetrical?
   b) Use Excel to calculate the mean, standard deviation and skewness of the daily return percentages.
   c) Use the mean and standard deviation you found in part (b) to compute the probability that the market will drop 2% or more on a given day assuming the daily return percentages are normally distributed.
      d) Use the historical data to determine the fraction of days the market fell by 2% or more. Compare your result with your calculation in part (c). What do you conclude about the normality of the daily return percentages?

Winter 2019 MICA II Assignment #3: Bifurcations in Chaotic Systems

Task guidelines created by Bill Ralph (Brock University)

In this assignment you will create and study the amazing bifurcation diagram for the logistic map.

1) a) Put four textboxes, a picture box, a "Show Orbit" button, a "Show Limit" button and a quit button on a form.
   b) Name the textboxes txtSeed, txtBound, txtLeft and txtRight and make their initial values on the form equal to 0.2111111, 10, 0 and 4 respectively. Label these textboxes so we know what they're for.
   c) Make the picture box 500 by 500 pixels.
   d) In the questions that follow, assume that your picture box runs from txtLeft to txtRight along the horizontal axis and from 0 up to 1 on the vertical axis. (The code for the rescaling of the x -axis is given below.)

2) We are going to plot the ***orbits*** of f(x) = kx(1-x) for k between txtLeft and txtRight.

   a) In your code, define doubles xL, xR , k and Seed. Also define an integer i.
   b) Set xL=txtLeft , xR=txtRight and Seed = txtSeed
   c) Set k=xL+i*(xR-xL)/500
      (As i goes from 0 to 500, what are the largest and smallest values of k?)
   d) <u>The Show Orbit Button</u>
      As i runs from 0 to 500. plot all of the points (i,Seed), (i,f(Seed)), (i,f(f(Seed))), ... up to the number in txtBound using the corresponding k value from (c)..
   e) <u>The Show Limit Button</u>
      Same as (d) but don't plot the first n values. Experiment with n until you get a sharp picture. How sensitive is the plot to changes in the Seed value?

3) Identify the next six points after k=3 where bifurcations occur.

4) Based on your results in (3), give an estimate for the k value where the system first becomes chaotic.

5) For k= 3.3, use Maple to find all fixed points of fof(x). Explain where these points appear in your plots. Find $\left|\frac{d}{dx} fof(x)\right|$ at each of the fixed points. Interpret the result. Hand in all of your calculations and conclusions.

6) For k= 3.5, use Maple to find all fixed points of fofofof(x). Explain where these points appear in your plots.

7) Find something interesting about the bifurcation diagram that I don't know.

Winter 2019 MICA II Assignment #4: Battle Simulations

Task guidelines created by Bill Ralph (Brock University)

This assignment concerns the discrete Lanchester equations:

$$X_{n+1} = X_n - \mathbf{a}*Y_n$$
$$Y_{n+1} = Y_n - \mathbf{b}*X_n$$

Assume the time unit is one day, allow fractions of a soldier and decide that a battle is over when there is less than 1 person left in an army.

1) Write a program called "Battle Simulation" that allows a user to enter the initial size of the armies for X and Y and the values of **a** and **b**. At the click of a button labeled "Run Battle" we see the complete history of the battle appear in a text box. The output is neatly set up and labeled so that the days are numbered and we can see the size of each army on each day. (Note: you can stop battles after 1000 days.).

   a. Suppose that initially the X army has 2000 troops and the Y army has 3000 troops. If **a**=0.01 and **b**=0.05., use your program to determine how long this battle will last. Hand in the result.

2) Add a picture box that is 500x500 to your form in part (1). Suppose that initially the X army has 2000 troops and the Y army has 3000 troops. Imagine that the **b** values run from 0 up to .5 on the vertical axis and the **a** values run from 0 to .5 on the horizontal axis. At the click of a button labeled "Battle Plot", plot a Blue dot at a point (**a**,**b**) where X wins in less than 11 days, a Green dot at a point (**a**,**b**) where Y wins in less than 11 days and a Black dot at a point (**a**,**b**) where the battle takes 11 or more days.

   a. Find an equation in terms of **a** and **b** for the line through the origin that is contained in the black region. Explain your method.

   b. How would you interpret the line in (b) in terms of the outcomes of battles?

   c. Find the **b** coordinates of the points on the curved boundary of the blue region where the **a** coordinates are 0.1 and 0.2. Assuming this boundary is given by $c\mathbf{a}^2 + d\mathbf{b}^2 = 1$, use these two points to estimate c and d. Hand in your calculations which may include Maple output.

3) Suppose we extend our model so that new troops are arriving at random every day as follows:

$$X_{n+1} = X_n - \mathbf{a}*Y_n + r*T$$
$$Y_{n+1} = Y_n - \mathbf{b}*X_n + s*100$$

In these equations, T is some positive integer that is fixed for the duration of the battle and r and s are uniform random numbers between 0 and 1 that can change every day. Suppose that initially the X army has 2000 troops and the Y army has 3000 troops. Also assume that **a**=0.01 and **b**=0.05.

a. Write a program called "Battle Arrivals" to investigate the effect of changing T on the probability that the X army wins in less than 20 days. Your program should generate a graph of the probability the X army wins plotted against T.

b. Estimate the value of T for which the probability that the X army wins in less than 20 days is 0.8.

Hand in your program and a written summary of your findings and conclusions.

Hand in your three working programs labeled "Battle Simulation", "Battle Plot" and "Battle Arrivals" and the answers to the questions raised in parts 1,2 and 3.

Hint: I used the following code to help me find the a and b coordinates of points in the picturebox:

```
  Private Sub PictureBox1_MouseDown(ByVal sender As Object, ByVal e As
System.Windows.Forms.MouseEventArgs) Handles PictureBox1.MouseDown
      TextBox6.Text = e.X / 1000
      TextBox7.Text = (500 - e.Y) / 1000
   End Sub
```

Winter 2019 MICA II Assignment #5: Original Final Project

Task guidelines created by Bill Ralph (Brock University)

1) The project must be done by 2 people and approved by me before you start. Note that you are to supply a checklist of features.

2) The project can by one of three types which are all considered to be equally important:
   a) an **INVESTIGATION** of a mathematical problem
   b) a **LEARNING OBJECT** designed to teach/test a mathematical concept. (Must be designed for a particular High School or Brock mathematics course.)
   c) a “real world” **APPLICATION** of mathematics

3) The project should demonstrate sophisticated knowledge of vb.net.

4) The interface must be very friendly, self-explanatory and a pleasure to work with. In the case of a Learning Object, the interface will be given more weight in the grading.

Your write-up will have the following information under the indicated headings:

A) **PROJECT TYPE**
State whether your program is an Investigation, a Learning Object or an Application.

B) **TARGET AUDIENCE**
Describe the group of people who would benefit from working with your project. In the case of a Learning Object, state the mathematics course for which it was designed.

C) **PURPOSE AND BACKGROUND**
State the overall mathematical or pedagogical purpose of your project and provide the necessary background.

D) **CHECKLIST OF FEATURES**
Provide a checklist of 5 special features of your program for which you should be given credit.

E) **SUMMARY OF OBSERVATIONS**
In the case of an Investigation or an Application, give a coherent summary of what you did and what you observed. For a Learning Object, design a short survey and have it completed by at least two university students who worked with your program. Hand in the completed surveys and your response to the surveys.

Math 2P40 Final Project Grading

Name________________________Id___________________________

Name________________________Id___________________________

Interface Design- Attractivenss, Effectiveness

Teaching	/40	Investigation	/20	Application	/20

Concept and Originality

Teaching	/20	Investigation	/30	Application	/30

Programming Sophistication

Teaching	/20	Investigation	/30	Application	/30

Writeup/Results

Teaching	/20	Investigation	/20	Application	/20

Final Grade:______________