Mathematics Integrated with Computers and Applications III*

An Undergraduate Programming-Based Mathematics Course for Prospective Teachers, Brock University (Canada)

Five Programming-Based Math Project Assignments

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This is part of a newly designed course that is built on the former full-year *Mathematics Integrated with Computers and Applications II-III* (MATH 2F40) course.

Since 2001, mathematics majors and prospective mathematics teachers learn through a sequence of three courses (MICA I-II-III) to design, program and use interactive enviroments for the investigation of math concepts, conjectures, theorems and applications. The MICA III course was recently divided into two sections; one for mathematics and science majors (MICA III) and one for prospective mathematics teachers (MICA III*), and both new courses were offered for the first time in Winter 2017.

This material is planned to be further adapted in the next offering (Winter 2018).

Programming Languages:

- *Visual Basic.NET* through *Visual Studio* development environment (this is the same as in MICA I-II)
- Scratch

For some information about the MICA courses, see:

- Ralph, W. (2001). Mathematics takes an exciting new direction with MICA program. *Brock Teaching,* 1(1), 1.
- Buteau, Muller, & Ralph (2015). Integration of Programming in the Undergraduate Mathematics Program at Brock University. In Online Proceedings of the *Math + Coding Symposium,* London (Canada), June 2015.

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MICA III* Assignment #1: The Calendar (Intro to Programming Scratch)

Task guidelines created by *Chantal Buteau*

To be solved individually in Scratch:

> January 1, 1600 fell on a Saturday. Write a code in Scratch where the input is the date (day, month, year) after 1 January 1600, and the output is the day of the week.[1]

Then reflect on your solving process. Briefly comment on:

i) Key moments or strategies used when completing your project
ii) Each, *if applicable*, of the 7 affordances (from Gadanidis et al., 2016) of CT-based activities in math learning
iii) Your engagement in CT (according to Brennan and Resnick's (2014) three-dimensional framework of CT)

What to submit:

The main part of your assignment (85%): It consists of your working program in Scratch submitted on a USB, and a list of dates with week days that you used/tested to ensure that your calendar works properly.

The second part of your assignment (15%): It consists of your written reflection (up to 1 page long):

i) Problem solving strategy
ii) Affordances of CT-based activities in math learning (using Gadanidis et al., 2016)
iii) Engagement in CT (using Brennan and Resnick, 2014)

REFERENCES

Gadanidis, G., Hughes, J. M., Minniti, L., & White, B. J. (2016). Computational thinking, grade 1 students and the binomial theorem. *Digital Experiences in Mathematics Education*, 1-20.

Brennan, K., & Resnick, M. (2012, April). New frameworks for studying and assessing the development of computational thinking. In *Proceedings of the 2012 annual meeting of the American Educational Research Association, Vancouver, Canada* (pp. 1-25).

[1] Problem proposed at CMESG 2016 by Miroslav Lovric (McMaster University, Canada)

MICA III* Assignment #2: Needles on the Floor

Task guidelines originally created by *Bill Ralph* and adapted by *Chantal Buteau*

A) Consider the following problem:

> A needle of length 1/2 is dropped onto a floor of parallel slates that are 1 unit apart. Find the probability that the needle touches a line.

In vb.net, create a simulation of this problem with two visual representations in order to estimate the probability:

i) Needles of length 1/2 falling on a [0,2] x [0,2] plane with parallel lines (at y=1/2 and y=3/2)whereby one can assume the middle point of the needle falls within the parallel line strip; show in red the needles that touch a line, and show the other needles in blue.

ii) Use a Monte Carlo method to estimate an appropriated area corresponding to the desired probability.

Call your program 'NeedlesOnParallelSlatedFloor'. The user's input is the number of needles dropped (i.e., # of trials). The program should display, in addition to i) and ii), the empirical probability (in % format). It should also display your approximation of π.

B) Now consider the following problem:

> A needle of length 1 is dropped onto a floor made of unit square tiles. Find the probability that the needle touches any of the lines.

In vb.net, create a simulation of this problem with two visual representations in order to estimate the probability:

i) Needles falling on a [0,2] x [0,2] plane with parallel lines (at y=1/2 and y=3/2 and at x=1/2 and x=3/2); show in red the needles that touch a line, and show the other needles in blue.

ii) Use a Monte Carlo method to estimate an appropriated area corresponding to the desired probability.

Call your program 'NeedlesOnSquareTiledFloor'.

Reflection: Reflect on your learning experience. Identify (and briefly justify) all of the computational practice activities (according to Weintrop et al., 2016) that you engaged in as you completed tasks A and B. In addition, describe 'to a virtual Grade10' the mathematical ideas or concepts of one or both tasks.

What to submit:

The main part of your assignment (85%): It consists of your two working programs in vb.net submitted on a USB. As well as a written description of your mathematical model used in Problem B.

The second part of your assignment (15%): It consists of i) the list of computational practice activities associated with tasks A and B, and a short justification for each (no more than one page); and ii) a description of mathematical ideas or concepts associated to either or both tasks A and B, in the format of your choice (e.g., a short written paragraph; a PowerPoint slide or slide show; a video; a poster; etc.).

REFERENCES:

Weintrop et al. (2016). Defining Computational Thinking for Mathematics and Science Classrooms. *Journal of Science Education and Technology*, *25* (1), 127-147

MICA III* Assignment #3: Chords in a Circle

Task guidelines created by *Chantal Buteau*

Consider the following problem:

> Consider a circle and an equilateral triangle inscribed in it. Pick a chord at random in the circle. What is the probability that the chord is longer than a side of the equilateral triangle?

Part A

In vb.net, create a simulation of this problem that allows you to estimate the desired theoretical probability. The program should visually show the statistical experiments (similar to i) in Assignment 2), as well as a numerical value of the calculated empirical probability.

Part B

Write a short description of the mathematics of the problem, as well as a short discussion of your results. Further elaborate your interface as to communicate (possibly with some visual parts) your results to a fictive Grade 10.

Reflection: Reflect on your learning experience; i) identify (and briefly justify) all of the computational practice activities (according to Weintrop et al., 2016) that you engaged in as you completed your task, and ii) reflect on your solving process by describing one of several key ideas/moments – could this problem be solved in a 'traditional' paper-pencil activity?

What to submit:

The main part of your assignment (88%): It consists of your working program in vb.net, and elaborated interface (Part B) submitted on a USB. It also includes a written description of your mathematical model (Part B).

The second part of your assignment (12%): It consists of the list of computational practice activities associated with your task and a short justification for each (no more than one page), plus your reflection on your learning experience (solving process – no more than ½-page).

REFERENCES:

Weintrop et al. (2016). Defining Computational Thinking for Mathematics and Science Classrooms. *Journal of Science Education and Technology*, *25* (1), 127-147

MICA III* Assignment #4: Prey-Predator Model

Task guidelines originally created by *Bill Ralph* and adapted by *Chantal Buteau*

This assignment concerns an exploration, by use of graphical analysis, of the predator-prey model described by the Lotka-Volterra equations, namely:

$$\frac{dR}{dt} = kR - aRW$$

$$\frac{dW}{dt} = -rR + bRW$$

where a, b, k, and r are positive constants, t is time in months, and R (for 'rabbits') represents the prey population and W (for 'wolves') represents the predator population. As well of its modified version as:

$$\frac{dR}{dt} = kR(1 - \frac{1}{c}R) - aRW$$

$$\frac{dW}{dt} = -rR + bRW$$

where c is a positive constant.

Part A – Your program in vb.net

Modify Euler's method in order to solve numerically either of the two systems of equations by use of a 3-dimensional dynamical system involving t_n, R_n, and W_n, and display i) the numerical solution as graphs (in the same plot but with two different scaling for the vertical axis) of the pair R(t) and W(t), as well as ii) the phase trajectory for initial conditions given by a mouse click in the trajectory plane. The following gives more details about the programming of your interactive environment:

- Your interface will contain two picture boxes – give them descriptive names (e.g. PopulationGraphs and TrajectoryPlane).
- Your interface will also contain many textboxes for all the parameters involved in the model and in the graphical representations of the solutions. For example, a textbox for *MaxR*, the maximum number of prey for the vertical axis of your prey population graph – use a descriptive name for your textbox, e.g., txtMaxR.
- Your interface will contain only two buttons; one named 'PlotPopulations' and a second one named 'Exit'. It will also contain a checkbox labeled 'modified model' – when checked, the program will solve the modified system of equations.
- The graphs of the two population functions R(t) and W(t) will be shown in a same plot using two different colors; the horizontal axis (for time t) will use the same scale for both graphs, but the vertical axis will use different scales (from 0 to MaxR, and from 0 to MaxW) for the two population functions. Add a legend for your colors on the form.
- The phase trajectory will be plotted for initial conditions (R_0, W_0) given by clicking in the picture box. In other words, when the user clicks on a point in the picture box, we see the phase trajectory drawn from that point.

  The following code may be of help - it displays the x and y (math) coordinates (in [0,MaxX] x [0,MaxY]) of the point selected by a mouse click in the picturebox1 (of size 300 x 400):

```
Private Sub PictureBox1_MouseDown(ByVal sender As Object, ByVal e As System.Windows.Forms.MouseEventArgs) Handles
PictureBox1.MouseDown
        TextBox1.Text = (e.X / 300) * MaxX
                'e.X reads the vb.net x value, and converts it into math x-value in [0,MaxX]
        Textbox2.Text = (1 - e.Y / 400) * MaxY
                'e.Y reads the vb.net y value, and converts it into math y-value in [0,MaxY]
End Sub
```

Part B – Your exploration using your interactive environment

Using your interactive environment, answer the following questions (and provide screen shots to support your arguments) about the model of aphids (prey) and ladybugs (predator) whereby we set k=2, a=0.01, r=0.5, and b=0.0001. We also set initial conditions to R_0=1000 and W_0 = 200. For graphing purposes, you may want to set MaxR=15000, MaxW=400, and h=0.001.

1. Plot the population of both species, and describe how they evolve over time.
2. Determine the equilibrium points, describe what it means in the context of the modeled situation, and categorize them based on an exploration of the trajectories close to the equilibrium points: stable, unstable, or neither (i.e. periodic motions around the point).
3. Describe how the populations change when k is increased by a small amount and explain why you would expect the model to behave this way.
4. Describe how the populations change when a is increased by a small amount and explain why you would expect the model to behave this way.
5. What is the interval, in months, between peak aphid populations? Show how you obtained your answer.
6. If we were to now consider the modified model of these populations with c=10,000:
    a) Describe, in the absence of ladybugs, what the model would predict about aphids?
    b) How would you describe what c represents in the model?
    c) Describe, according to this model, how the populations evolve over time and how it differs from the initial model in questions 1-2.

Part C – Reflection

Reflect on your learning experience by identifying (and briefly justifying) all of the computational practice activities specific to a math/science classroom (according to Weintrop et al., 2016) that you engaged in as you completed your Lotka-Volterra project. Also, reflect on where you stand as a "computational thinker": skills/knowledge you have developed thus far, and what you feel you still need to (further) learn or develop (you may wish to use, in particular, Brennan & Resnick's (2014) 3-dimensional framework to guide your reflection).

What to submit:

The main part of your assignment (83%): It consists of your working program in vb.net submitted on a USB, as well as a written report of your answers to Part B.

The second part of your assignment (17%): It consists of the list of computational practice activities associated with your task, and a short justification for each (no more than one page). It also consists of your reflection as an accomplished "computational thinker" (about one page).

REFERENCES:

Weintrop et al. (2016). Defining Computational Thinking for Mathematics and Science Classrooms. *Journal of Science Education and Technology*, *25* (1), 127-147

Brennan, K., & Resnick, M. (2012, April). New frameworks for studying and assessing the development of computational thinking. In *Proceedings of the 2012 annual meeting of the American Educational Research Association, Vancouver, Canada* (pp. 1-25).

MICA III* Assignment #5: DNA, The Secret of Life!

Task guidelines originally created by *Bill Ralph* and adapted by *Chantal Buteau*

This assignment is an introduction to the important and growing new field of bioinformatics which uses mathematics to analyze biological data. Your task is to create and use a program that allows you to explore the DNA content of a living being's genome.

Part A – The program

Create a program in vb.net that calculates diverse proportions in a DNA sequence: proportions of each nucleotide (G, C, T, and A), C+G content (called the *base composition*), the paired nucleotides CG, and the trinucleotides TAA, called a stop codon. From a text file, the program will read the sequence (in the format found under 'Origin' at http://www.ncbi.nlm.nih.gov/nuccore/7768734), display it and its size in a textbox, and will 'clean it' (i.e., remove spaces and numbers) before calculating the proportions.

The program will also allow a visualization of the DNA sequence as a kind of normalized walk in the plane: Start at (0,0); and read through the DNA sequence; if 'C': $+1/n_C$ in x-direction; if 'G': $-1/n_G$ in x-direction; if 'T': $+1/n_T$ in y-direction; and if 'A': $-1/n_A$ in y-direction, e.g. where n_A is the number of A in the DNA sequence. When a checkbox is checked, it will display in the same plane (using a different color) a normalized random walk of same length.

More about the DNA sequence files: Go to http://www.ncbi.nlm.nih.gov/nuccore/7768734. Either use this data, or type in the name of any living thing in the search box at the top, then scroll down and copy the small amount of DNA that is under the word "Origin". Paste this sample including the numbers and spaces into notepad and save it. This file should appear on your computer as FileName.txt.
HINT: I added a 'OpenFileDialog1' object (shown at the bottom) on my form and used the following code to select the file and read the DNA sequences from it:

```
Dim AllText as String
OpenFileDialog1.Filter = "Text files (*.txt)|*.txt"

'display the open dialog box
If OpenFileDialog1.ShowDialog() = Windows.Forms.DialogResult.OK Then
        Try 'open file and trap any errors using handlers
            AllText = My.Computer.FileSystem.ReadAllText(OpenFileDialog1.FileName)
                'read the whole content of the textfile selected by the user, and save 'it as a string
                in AllText
        Catch ex As Exception
                 MsgBox("An error occured" & vbCrLf & ex.Message)
        End Try
End If
```

Part B – The investigation

Use your program to answer the following questions in a written report.
1) Use the ***Human DNA sequence*** from: https://www.ncbi.nlm.nih.gov/nuccore/AC073210.8. We will first question the independence of the nucleotides G and C present in the DNA sequence by exploring the 'CG' content. In the following, use the relative frequency of each nucleotide as probability that a nucleotide selected at random is of that type.

i) Compare the proportion of 'CG' content with the (hand-calculated) conditional probability that a nucleotide chosen at random is G given that the previous one in the sequence is a C. What does it suggest: *that the nucleotide G and C in the DNA sequence are occurring randomly and independently?* Explain.

ii) Conduct a statistical test on proportions to question whether the nucleotides C and G are independent by determining whether the observed proportion of 'CG' in the DNA sequence is significantly different from the expected proportion if C and G were occurring randomly and independently. *What do you conclude?*

iii) Have also a look at the stop codon TAA. Does it occur more or less often than you would expect by chance if you assume that the nucleotides are distributed randomly and independently (again based on the relative frequencies of individual nucleotides)? Explain.

HINT: Suppose for example, that your DNA string was catggacat. Then the two letter strings would be ca, at, tg, gg, ga, ac, ca and at so the probability of picking the two letter string ca at random would be 2/8.

2) Look at the ***DNA sequence of five other living beings***.

i) Compare the proportions of their nucleotides, and of their base composition (G+C content). Summarize the results (remember to indicate the name of the living being) and comment on whether or not these proportions are more or less constant for the different types of life you have examined.

ii) Research showed that the frequency of occurrence of TAA stop codon is correlated to the GC-content. What does your data suggest: a positive or negative correlation? Justify you answer. Write a sentence (to a fictive Grade 10 student) that describes what this means.
BONUS: add a visualization of the correlation in your vb.net code.

iii) For each living being, generate their "DNA sequence normalized walk" visualization. Compare them; *do they seem different to you?* Provide screenshots.

iv) *Do the DNA sequence normalized walks in ii) look random to you?* Compare them with normalized random walks of the same length. Provide screenshots and comment.

PART C – The reflection

Reflect on your learning experience by identifying (and briefly justifying) all of the computational practice activities specific to a math/science classroom (according to Weintrop et al., 2016) that you engaged in as you completed your DNA sequence project. Also, reflect on the CT aspect or part of this assignment that you found most challenging– why was it challenging to you?

Since it's the last assignment, I also want you to reflect on the core of this course: CT in/for mathematics learning. Explain to a fictive Grade 10 student what it is and/or why it is important (choose the media you want! E.g. write a short paragraph, create a PowerPoint, a video, a poster, etc.). You may wish to look at http://researchideas.ca (in particular the different issues of the Math + Code'Zine published at http://researchideas.ca/mc/).

What to submit:

The main part of your assignment (84%): It consists of your working program in vb.net submitted on a USB, as well as a written report of your answers to Part B.

The second part of your assignment: It consists of the list of computational practice activities associated with your task, a short justification of each (no more than one page), as well as your reflection on the CT aspect or part of the assignment that was most challenging (no more than ¾ page) – (6%). It also consists of your description of what CT in/for mathematics learning is to a fictive Grade 10 student in the media of your choice (10%).

REFERENCES:

Weintrop et al. (2016). Defining Computational Thinking for Mathematics and Science Classrooms. *Journal of Science Education and Technology*, *25* (1), 127-147